

Consequences of a Z_2 Symmetry for Neutrino Oscillations

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Abstract

A $Z_{2L} \times Z_{2R}$ generation symmetry in the neutrino sector predicts the atmospheric neutrino mixing to be maximal, and the MNS matrix element U_{e3} to be zero, consistent with observations. Solar neutrino mixing may be maximal but is not required by the symmetry. Neutrino masses of the first two generations are predicted to vanish, providing a first approximation to the oscillation data. The consequence of a smaller Z_2 symmetry is also discussed. In that case, deviation from the $Z_{2L} \times Z_{2R}$ result is of the order of the neutrino mass ratio between the first two generations and the third generation.

Right-handed neutrinos must be present to explain the deficit of solar and atmospheric neutrinos through oscillations. These are unique probes at high energy for they have no standard-model quantum numbers to allow a pollution by standard-model interactions. Recent results from Super-Kamiokande suggest that the mixing for solar neutrinos and the atmospheric neutrinos are both maximal, and the $(\Delta m)^2$ for solar neutrino oscillation is much smaller than the $(\Delta m)^2$ for atmospheric neutrino oscillations [1]. The negative result of CHOOZ [2] also limits the magnitude of the MNS mixing matrix [3] element $|U_{e3}|^2$ to be smaller than 0.02 to 0.047, depending on the exact value of the atmospheric neutrino mass difference. Is there a simple way to understand these facts? Many models have been proposed [4]. We suggest that a $Z_{2L} \times Z_{2R}$ generation symmetry in the sea-saw scenario

will naturally explain many of these observed results. By Z_2 we mean a finite group with elements $\{e, g\}$ so that g^2 is the identity element e . Specifically, under such a symmetry, the atmospheric neutrino mixing is maximal, and the MNS matrix element $U_{13} \equiv U_{e3}$ is zero. The neutrino masses of the first two generations vanish, which may be a fair approximation to Nature since the mass difference observed in atmospheric neutrino oscillation is much larger than that seen in solar neutrino oscillation. Under such a symmetry, the solar neutrino mixing may be maximal but is not required to be so.

Let D be the Dirac mass matrix for neutrinos in the flavor basis, and M the Majorana mass matrix in the same basis for right-handed neutrinos. We shall assume oscillations to occur only among the active neutrinos, so both are 3×3 complex matrices, although M is symmetrical. When the right-handed neutrinos are integrated out, the effective mass matrix for the left-handed neutrinos in the flavor basis is [5]

$$m' = D^T M^{-1} D. \quad (1)$$

To diagonalize it, we must use the MNS matrix U to rotate the flavor basis into the energy eigenbasis, then

$$m' = U^* m U^\dagger, \quad (2)$$

with $m = \text{diag}(m_1, m_2, m_3)$.

Let ν_{aL} and ν_{aR} be respectively the left- and right-handed neutrino field of the a th generation. Assume the mass terms in the neutrino Lagrangian to be invariant under the $Z_{2L} \times Z_{2R}$ transformation $\nu_{2A} \rightarrow -\nu_{3A}$, $\nu_{3A} \rightarrow -\nu_{2A}$, and $\nu_{1A} \rightarrow -\nu_{1A}$, where A is either L or R . Then the matrices D and M^{-1} must be of the form

$$D = m_D \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}, \quad M^{-1} = m_M^{-1} \begin{pmatrix} \mu_1 & \mu_2 & \mu_2 \\ \mu_2 & 1 & \mu_3 \\ \mu_2 & \mu_3 & 1 \end{pmatrix}. \quad (3)$$

Using (1), this leads to

$$m' = \frac{2m_D^2(1 - \mu_3)}{m_M} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}. \quad (4)$$

Other than the common scale factor which may be complex, the matrix is real and symmetrical, so it can be diagonalized by a real orthogonal transformation. The eigenvalues are $(0, 0, 1)$, but the matrix U used for diagonalization is not unique because of degeneracy of the first two eigenvalues. For the purpose of later generalization it is useful to deduce these results explicitly from

$$\begin{aligned} (m)_{ij} &= U_{ai}(m')_{ab}U_{bj} \\ &= \frac{2m_D^2(1 - \mu_3)}{m_M} (U_{2i} - U_{3i})(U_{2j} - U_{3j}). \end{aligned} \quad (5)$$

In order for m to be diagonal, two of the following three combinations must vanish: $U_{21} - U_{31}$, $U_{22} - U_{32}$, and $U_{23} - U_{33}$. Let us first consider the case when $U_{23} - U_{33} \neq 0$, but the other two combinations vanish. In that case $m_1 = m_2 = 0$ and $m_3 = (2m_D^2(1 - \mu_3)/m_M)(U_{23} - U_{33})^2$, thus giving rise to the extreme limit of a normal hierarchy for the neutrino masses, which is a good approximation to reality because $(\Delta m)_{12}^2 \ll (\Delta m)_{23}^2$. To simplify writing let $a = U_{21} = U_{31}$, $b = U_{22} = U_{32}$. Then U is of the form

$$U = \begin{pmatrix} c & d & e \\ a & b & f \\ a & b & g \end{pmatrix}. \quad (6)$$

We will adopt the usual phase convention so that c, d, f and g are real, and the imaginary part of a and of b are proportional to that of e . Normalization of the second and third rows of the matrix U requires $|a|^2 + |b|^2 + f^2 = 1 = |a|^2 + |b|^2 + g^2$. Hence f and g have the same magnitude. Orthogonality of the second and the third rows of U then requires $f = -g$ and $f^2 = |a|^2 + |b|^2 = \frac{1}{2}$. In other words, atmospheric neutrino mixing is *maximal*, consistent with the Super-Kamiokande observation. Using the fact that the first row of U must be orthogonal to both the second and the third rows, we conclude that $e = 0$, consistent with

the CHOOZ observation. In that case there will be no CP violation observable through neutrino oscillations. With the present phase convention both a and b become real, and the magnitudes of a, b, c, d are related by unitarity. So, there is only one free parameter θ_{12} left in describing U . For $c = -d = 1/\sqrt{2}$, maximal mixing occurs in solar neutrino mixing, but this is not required to be so by the symmetry. In summary, judging from neutrino oscillations, the $Z_{2L} \times Z_{2R}$ symmetry appears to be a good approximation to Nature.

The other two solutions of (5) leads to either $m_1 \neq 0$ or $m_2 \neq 0$, with the other two diagonal matrix elements of m zero. The former case leads to $U_{11} = 0$, and the latter case leads to $U_{12} = 0$, neither is consistent with solar neutrino and CHOOZ observations. They will therefore be rejected.

Note also that the symmetry interchanges generations 2 and 3, with a minus sign. If instead it interchanged generations 1 and 2, which at first sight might seem a more reasonable thing to do in view of the mass hierarchy, the result would be the same as above but with the first and third rows of the matrix U permuted. This would not be consistent with experiment.

If the charged lepton and the quark Dirac mass matrices are subject to the same symmetry, then all of them would have a mass spectrum proportional to (0,0,1), not a bad first approximation. In that case the CKM matrix would be $\mathbf{1}$, again a reasonable first approximation given that the Cabibbo angle is small.

There is the question of how a small but non-zero value of $m_{1,2}$ can be obtained. One possibility is to have a small breaking of $Z_{2L} \times Z_{2R}$, into a diagonal Z_2 where the left-handed and the right-handed particles are simultaneously transformed. This does not alter the parametrization of M , but allows small parameters to occur in D where its present matrix elements are zero. It should be possible to tune these parameters to get finite masses for the first two generations. Since a different set of parameters may be present for the charged leptons, the up-type quarks, and the down-type quarks, it is quite conceivable that different mass spectra can be obtained for these different particles, while obtaining also a realistic CKM matrix.

Let us now explore this scenario more systematically. The Z_2 symmetry requires the (symmetric) matrix m' in (1) and (2) to obey $m'_{12} = m'_{13}$ and $m'_{22} = m'_{33}$. Using (2), these two requirements can be written as

$$\sum_i \alpha_i \beta_i m_i = 0, \quad (7)$$

$$\sum_i \gamma_i \beta_i m_i = 0, \quad (8)$$

where

$$\alpha_i = U_{1i}^*,$$

$$\beta_i = U_{2i}^* - U_{3i}^*,$$

$$\gamma_i = U_{2i}^* + U_{3i}^*,$$

thus giving rise to constraints on U controlled by the neutrino mass values. For the extreme hierarchy spectrum $(m_1, m_2, m_3) \propto (0, 0, 1) \equiv s_1$ discussed above, the constraints are $\alpha_3 \beta_3 = \gamma_3 \beta_3 = 0$. Being second order equations there are two solutions: (1a) $\alpha_3 = \gamma_3 = 0$, and (1b) $\beta_3 = 0$. Solution (1a) is the same as the $Z_{2L} \times Z_{2R}$ solution before. To see that, note that the third column of U is fixed by this constraint and unitarity to be $(U_{13}, U_{23}, U_{33}) = (0, f, -f)$, with $f^2 = \frac{1}{2}$. Orthogonality of the first two columns to the third then fixes U to be of the form (6), with parameters identical to those in $Z_{2L} \times Z_{2R}$ as per the arguments given below (6). Solution (1b) gives $U_{23} = -U_{33}$, which is not very restrictive. For a realistic hierarchical spectrum $s_2 = (\eta, \epsilon, 1)$, with $\epsilon \ll 1$ and η either of the same order or much less than ϵ , the constraints become $\alpha_3 \beta_3 = O(\epsilon)$ and $\gamma_3 \beta_3 = O(\epsilon)$. The two corresponding solutions are: (2a) $\alpha_3 = O(\epsilon)$, $\gamma_3 = O(\epsilon)$, and $\beta_3 = O(1)$, and (2b) $\beta_3 = O(\epsilon)$ and $\alpha_3, \gamma_3 = O(1)$. As before, solution (2b) is not very restrictive. For (2a), $|U_{e3}|$ is of order ϵ and the atmospheric neutrino oscillation is close to maximal, with deviation of the order of $\epsilon = m_2/m_3$. In this way a Z_2 symmetry with a realistic mass spectrum as an input gives rise to practically the same predictions as the larger $Z_{2L} \times Z_{2R}$ symmetry.

Whether a more realistic mass spectrum should be obtained by an explicit breaking of $Z_{2L} \times Z_{2R}$ like the one described above, by renormalization-group corrections, or by

something else, will be left to future investigations.

This research is supported in part by the Natural Sciences and Engineering Research Council of Canada, and the Fonds pour la formation de Chercheurs et l'Aide à la Recherche of Québec. I am indebted to T.K. Kuo, Greg Mahlon, Gary Shiu, and Tony Zee for stimulating discussions.

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